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A SEQUENCE OF PIECEWISE ORTHOGONAL POLYNOMIALS (II).(U)
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A SEQUENCE OF PIECEWISE ORTHOGONAL POLYNOMIALS (II)

Y. Y. Feng and D. X. Qi

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ABSTRACT

In this paper an orthonormal sequence of piecewise polynomials of degree k is given. We study the construction and sign-change properties of this sequence and consider the convergence of the corresponding Fourier series. The results generalize those obtained earlier for piecewise constant and piecewise linear functions.

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Key Words: polynomial, piecewise polynomial, Legendre polynomial, series expansion, orthonormal function.

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SIGNIFICANCE AND EXPLANATION

We previously presented a class of piecewise linear orthonormal functions U_i that are complete in $L_2[0,1]$, and pointed out that any continuous function can be expanded in terms of U_i in the sense of uniform convergence by group. This paper generalizes those results to the case of piecewise polynomials of degree k. We construct the sequence for $k > 1$, study sign-change properties, and consider the convergence of the corresponding Fourier series. It is then shown that such a sequence of piecewise polynomials generalizes both the Walsh function and the Legendre function.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

A SEQUENCE OF PIECEWISE ORTHOGONAL POLYNOMIALS (II)

Y. Y. Feng* and D. X. Qi**

1. An Orthonormal Sequence of Piecewise Polynomials.

In this section we study a general procedure for constructing a sequence of orthonormal polynomial functions. We use the following notations:

$$Z := \{0,1,2,\cdots\}, \qquad I_k := \{1,2,\cdots,k\},$$

$$O_n := \{1,3,5,\cdots,2n-1\}, \qquad E_n := \{0,2,4,\cdots,2n\},$$

$$\lfloor x \rfloor := \max\{n : \text{integer},\ n < x\},$$

$$\langle f,g\rangle := \int_0^1 f(x)g(x)dx\ .$$

Suppose that $\{U_i\}_{i=0}^{k}$ is a sequence of orthonormal polynomials defined on $[0,1]$, even or odd with respect to the point $x = 1/2$ and the degree of U_i is i. At first we give the following theorem.

<u>Theorem 1.</u> There exist exactly $k+1$ polynomials $Q_{k,i}(x)$ $(i \in I_{k+1})$ of degree k with the property that

$$U_{k,2}^{(i)}(x) := \begin{cases} Q_{k,i}(x)\ , & 0 < x < 1/2\ , \\ (-1)^{k+i}Q_{k,i}(1-x)\ , & 1/2 < x < 1\ , \end{cases} \qquad i \in I_{k+1} \tag{1.1}$$

satisfies

$$\langle U_{k,2}^{(i)}(x), x^j\rangle = 0\ , \qquad j \in I_k \cup \{0\}, \qquad i \in I_{k+1} \tag{1.2}$$

$$\langle U_{k,2}^{(i)}(x),\quad U_{k,2}^{(j)}(x)\rangle = \delta_{ij}, \qquad i,j \in I_{k+1} \tag{1.3}$$

with

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$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$$

<u>Proof.</u> Let $k = 2m+1$ for $m \in Z$. Let

$$V_{k,2}(x) := \begin{cases} Q_k(x), & 0 < x < 1/2, \\ Q_k(1-x), & 1/2 < x < 1, \end{cases}$$

$$\bar{V}_{k,2}(x) := \begin{cases} \bar{Q}_k(x), & 0 < x < 1/2, \\ -\bar{Q}_k(1-x), & 1/2 < x < 1, \end{cases}$$

where Q_k, $\bar{Q}_k$ are polynomials of exact degree k, with leading coefficient 1. Because $V_{k,2}(x)$ is even and $\bar{V}_{k,2}(x)$ is odd with respect to $x = 1/2$, it is obvious that

$$\langle V_{k,2}, U_j\rangle = 0, \quad j \in O_{m+1}; \qquad \langle \bar{V}_{k,2}, U_j\rangle = 0, \quad j \in E_m.$$

From

$$\langle V_{k,2}, U_j\rangle = 2\int_0^{1/2} Q_k(x)U_j(x)\,dx = 0, \quad j \in E_m$$

we may get at least $m+1$ polynomials $Q_k(x)$, named $Q_{k,i}(x)$ $(i \in O_{m+1})$, of degree k which are linear independent in $[0, 1/2[$. The same kind of argument shows that there exist at least $m+1$ polynomials $\bar{Q}_k(x)$, named $Q_{k,i}(x)$ $(i \in E_m)$, of degree k which are linear independent in $[0, 1/2[$ and satisfy

$$\langle \bar{V}_{k,2}, U_j\rangle = 2\int_0^{1/2} \bar{Q}_k(x)U_j(x)\,dx = 0, \quad j \in O_{m+1}.$$

Using the process of orthogonalization, without loss of generality, we may suppose $\sqrt{2}\,Q_{k,i}(x)$ $(i \in E_m$ or $i \in O_{m+1})$ are orthonormal to each other in $[0, 1/2[$, i.e.

$$\int_0^{1/2} Q_{k,i}(x)Q_{k,j}(x)\,dx = 1/2\,\delta_{ij}, \quad i,j \in E_m \quad \text{or} \quad i,j \in O_{m+1}.$$

Let

$$U_{k,2}^{(i)} := \begin{cases} Q_{k,i}(x), & 0 < x < 1/2, \\ (-1)^{i+1} Q_{k,i}(1-x), & 1/2 < x < 1, \end{cases} \quad i \in I_{k+1}.$$

It is easy to check that $U_{k,2}^{(i)}$ satisfies (1.2) and (1.3).

Let

$$M_{2(k+1)} := \mathrm{span}\{U_0, U_1, \cdots, U_k, U_{k,2}^{(1)}, \cdots, U_{k,2}^{(k+1)}\}. \tag{1.4}$$

We denote the collection of all piecewise polynomials of order $k+1$ with partition Δ_n by $\mathbb{P}_{k+1,\Delta_n}$, where Δ_n is the uniform partition on 2^{n-1} intervals. It is obvious that

$$\dim \mathbb{P}_{k+1,\Delta_n} = (k+1)2^{n-1}.$$

From (1.4) we know

$$M_{2(k+1)} = \mathbb{P}_{k+1,\Delta_2}$$

since $\dim M_{2(K+1)} = \dim \mathbb{P}_{k+1,\Delta_2}$ and $M_{2(k+1)} \subseteq \mathbb{P}_{k,\Delta_2}$. Therefore the number of polynomials $Q_{k,i}$ do no more than $k+1$. We have proved the theorem for $k = 2m+1$. When k is even, the same kind of argument confirms the theorem. |||

There are many methods for constructing the $Q_{k,i}$ and thereby the $U_{k,2}^{(i)}$ $(i \in I_{k+1})$. We now show how to do this so that $U_{k,2}^{(i)}$ satisfies some smoothness requirements at the point $x = 1/2$.

Let

$$Q_{k,i}(x) := \sum_{j=0}^{k} a_j^{(i)} x^j \tag{1.5}$$

on $[0, 1/2]$ with $a_k^{(i)} = 1$.

For $k = 2m$, the coefficients $a_0^{(i)}, a_1^{(i)}, \cdots, a_{2m-1}^{(i)}$ $(i \in I_{2m-1})$ are defined by the following equations

$$\begin{cases} \langle U_{2m,2}^{(2i+1)}, \quad x^j \rangle = 0 , & j \in O_m, \\ \langle U_{2m,2}^{(2i+1)}, U_{2m,2}^{(j)} \rangle = 0 , & j \in O_i , \quad i \in I_m \cup \{0\}, \\ \dfrac{d^j Q_{2m,i}}{dx^j}\Big|_{x=1/2} = 0, & j \in E_{m-i-1}, \end{cases} \tag{1.6}$$

with $O_0 = \emptyset$, $E_{-1} = \emptyset$,

$$\begin{cases} \langle U_{2m,2}^{(2i)}, x^j \rangle = 0, & j \in E_m , \\ \langle U_{2m,2}^{(2i)}, \quad U_{2m,2}^{(j)} \rangle = 0 , & j \in E_{i-1} \setminus \{0\} , \quad i \in I_m, \\ \dfrac{d^j Q_{2m,2}}{dx^j}\Big|_{x=1/2} = 0 , & j \in O_{m-i}. \end{cases} \tag{1.7}$$

If $k = 2m+1$, the $a_0^{(i)}, a_1^{(i)}, \cdots, a_{2m}^{(i)}$ $(i \in I_{2m})$ are defined by the following equations

$$\begin{cases} \langle U_{2m+1,2}^{(2i+1)}, \quad x^j \rangle = 0 , & j \in E_m, \\ \langle U_{2m+1,2}^{(2i+1)}, U_{2m+1,2}^{(j)} \rangle = 0 , & j \in O_i , \quad i \in I_m \cup \{0\} , \\ \dfrac{d^j Q_{2m+1,i}}{dx^j}\Big|_{x=1/2} = 0 , & j \in O_{m-i}, \end{cases} \tag{1.8}$$

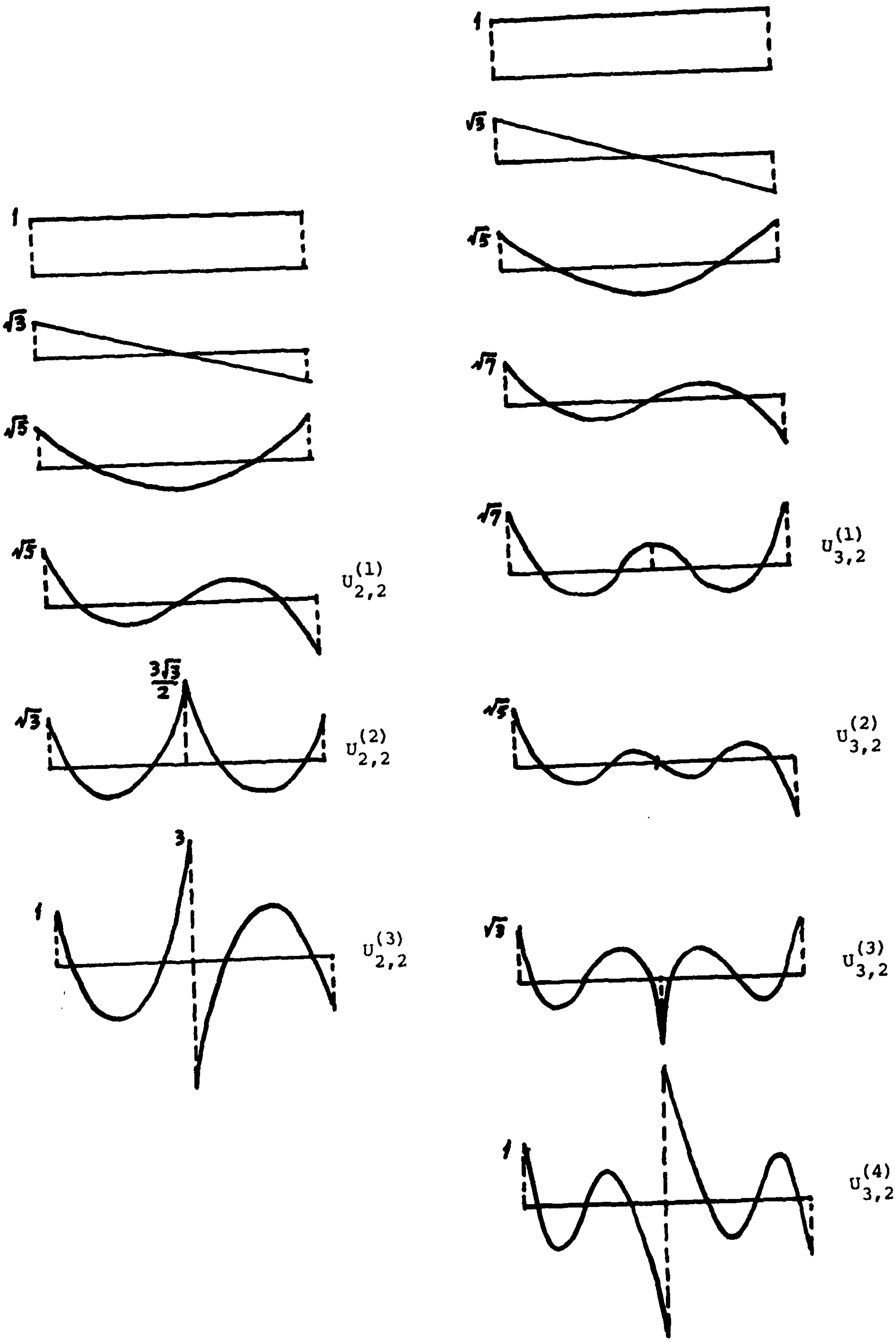
$$\begin{cases} \langle U^{(2i)}_{2m+1,2}, x^j\rangle = 0, & j \in O_{m+1}, \\ \langle U^{(2i)}_{2m+1,2}, U^{(j)}_{2m+1,2}\rangle = 0, & j \in E_{i-1}\setminus\{0\}, \quad i \in I_{m+1} \\ \left.\dfrac{d^j Q_{2m+1,i}}{dx^j}\right|_{x=\frac{1}{2}} = 0, & j \in E_{m-i}. \end{cases} \tag{1.9}$$

Equation systems (1.6), (1.7) and (1.8), (1.9) define uniquely $U^{(i)}_{k,2}$ $(i \in I_{k+1})$ respectively for k even and odd. When $k=2,3$, $U^{(i)}_{2,2}$ $(i \in I_3)$ and $U^{(i)}_{3,2}$ $(i \in I_4)$ are as follows after normalization:

$$\begin{cases} U^{(1)}_{2,2} = \sqrt{5}\,(16x^2 - 10x+1), \\ U^{(2)}_{2,2} = \sqrt{3}\,(30x^2 - 14x+1), \\ U^{(3)}_{2,2} = 40x^2 - 16x+1; \end{cases}$$

$$\begin{cases} U^{(1)}_{3,2} = \sqrt{7}\,(-64x^3 + 66x^2 - 18x+1), \\ U^{(2)}_{3,2} = \sqrt{5}\,(-140x^3 + 144x^2 - 24x+1), \\ U^{(3)}_{3,2} = \sqrt{3}\,(-224x^3 + 156x^2 - 28x+1), \\ U^{(4)}_{3,2} = -280x^3 + 180x^2 - 30x+1. \end{cases}$$

The graphs of these functions are given below.



1
$\sqrt{3}$
$\sqrt{5}$
$\sqrt{5}$
$U^{(1)}_{2,2}$
$\frac{3\sqrt{3}}{2}$
$\sqrt{3}$
$U^{(2)}_{2,2}$
3
1
$U^{(3)}_{2,2}$
1
$\sqrt{3}$
$\sqrt{5}$
$\sqrt{7}$
$\sqrt{7}$
$U^{(1)}_{3,2}$
$\sqrt{5}$
$U^{(2)}_{3,2}$
$\sqrt{3}$
$U^{(3)}_{3,2}$
1
$U^{(4)}_{3,2}$

After getting $U_{k,2}^{(i)}$ $(i \in I_{k+1})$, generally we define

$$U_{k,n+1}^{(2\ell-1)}(x) := \begin{cases} U_{k,n}^{(\ell)}(2x), & 0 < x < 1/2, \\ (-)^{k+\ell}\, U_{k,n}^{(\ell)}(2-2x), & 1/2 < x < 1, \end{cases} \tag{1.10}$$

$$U_{k,n+1}^{(2\ell)}(x) := \begin{cases} U_{k,n}^{(\ell)}(2x), & 0 < x < 1/2, \\ (-)^{k+\ell+1} U_{k,n}^{(\ell)}(2-2x), & 1/2 < x < 1, \end{cases} \tag{1.11}$$

$\ell \in I_{2^{n-2}(k+1)}$, $n \in Z \setminus \{0,1\}$.

We have the following theorem about the orthogonality of the sequence $\{U_{k,n}^{(i)}\}$.

Theorem 2. The sequence of functions $\{U_{k,n}^{(i)}\}$ is normal and orthogonal. I.e.

$$\langle U_{k,n}^{(i)}, U_{k,m}^{(j)}\rangle = \delta_{n,m}\delta_{i,j}$$

with $U_{k,1}^{(\ell+1)} := U_\ell$, $\ell \in I_k \cup \{0\}$; $i \in I_\mu$, $j \in I_\nu$ where

$\mu = (k+1)2^{\max(n-2,0)}$, $\nu = (k+1)2^{\max(m-2,0)}$.

Proof. The same kind of argument as in the proof of Theorem 1 in [3] confirms this theorem.

It is easy to see that

$$U_{k,m}^{(j)} \in P_{k+1,\Delta_n}, \quad m \in I_n, \ j \in I_\nu.$$

Let

$$M_{(k+1)2^{n-1}} := \mathrm{span}(U_0, U_1, \ldots, U_{k,n}^{(1)}, \ldots, U_{k,n}^{((k+1)2^{n-2})}).$$

It is obvious that

$$M_{2^{n-1}(k+1)} = P_{k+1,\Delta_n},$$

Therefore we have the following theorem.

<u>Theorem 3.</u> If f is a piecewise polynomial of degree k with breakpoints only at q/p, where q is integer and p is a power of two, then f can be exactly expressed by finite terms of the series $\sum_{i,j} \alpha_{i,j} U_{k,i}^{(j)}$.

2. Some Properties of the Sequence.

Let $S^+(a_0,\cdots,a_n)$ denote the maximum number of sign changes in the sequence $a_0,a_1,\cdots,a_n$ obtainable by giving any zero element the value $+1$ or -1, and define

$$S^-(f,[0,1]) := \sup\{n : \exists\, t_1 < t_2 < \cdots < t_{n+1},\ f(t_i)f(t_{i+1}) < 0\}$$

to be the number of strong sign changes of f on $[0,1]$.

Because $\{U_i\}$ $(i \in I_k \cup \{0\})$ is orthogonal on $[0,1]$, it is well known that

$$Z(U_i;[0,1]) = i\ , \qquad i \in I_k \cup \{0\}$$

with $Z(f;[a,b])$ denoting the number of zeros of f on $[a,b]$.

In order to study the sign changes of $U_{k,2}^{(i)}$ $(i \in I_{k+1})$ on $[0,1]$ we need the following lemma.

<u>Lemma 1 (de Boor[1]).</u> If $\underline{t} = (t_i)_1^{n+k}$ is nondecreasing in $[a,b]$, with $t_i < t_{i+k}$ all i, and $f \in L_1[a,b]$ is orthogonal to $\mathbf{S}_{k,\underline{t}}$ on $[a,b]$, then there exists $\underline{\xi} = (\xi_i)_1^{n+1}$ is strictly increasing in $[a,b]$ with $t_i \leq \xi_i \leq t_{i+k-1}$ (any equality holding iff $t_i = t_{i+k-1}$), $i \in I_{n+1}$, so that f is also orthogonal $\mathbf{S}_{1,\xi}$. Here $\mathbf{S}_{k,\underline{t}}$ denotes the collection of splines of order k with knot sequence $\underline{t}$.

In particular, if f is continuous, then it must vanish at the n points of some strictly increasing sequence $(\eta_i)_1^n$ with $t_i < \eta_i < t_{i+k}$ all i.

It is easy to see that

$$\mathbf{S}_{k+1,\Delta_2^{(i)}} = M_{k+1+i} = \mathrm{span}(U_0,U_1,\cdots,U_{k,2}^{(i)}),$$

where $\Delta_2^{(i)}$ is knot sequence $(t_j)_1^{2(k+1)+i}$,

$$t_j := \begin{cases} 0, & j \leq k+1, \\ 1/2, & k+1 < j \leq k+i+1, \\ 1, & j > k+2+i. \end{cases} \tag{2.1}$$

Using Lemma 1, we get

$$S^-(U_{k,2}^{(i+1)}) = k+1+i, \qquad i \in I_k \cup \{0\}, \tag{2.2}$$

since

$$\langle U_{k,2}^{(i+1)}, S\rangle = 0 , \quad S \in S_{k+1,\Delta_2^{(i)}}$$

and $U_{k,2}^{(i+1)} \in S_{k+1,\Delta_2^{(i+1)}}$.

We would like to study some further properties of piecewise polynomials $\{U_{k,2}^{(i)}\}$. At first, from the Budan-Fourier theorem ([4]), we know that if P is a polynomial of exact degree k , then

$$Z(P;(a,b)) \le S^-(P(a),\cdots,P^{(k)}(a)) - S^+(P(b),\cdots,P^{(k)}(b)) . \tag{2.3}$$

For convenience, suppose $k = 2m$, from (1.1), (2.2) we know

$$Z(Q_{k,i}; (0, 1/2)) = m + \left\lfloor \frac{i}{2} \right\rfloor. \tag{2.4}$$

By (1.6), (1.7)

$$S^+(Q_{k,i}(1/2), Q'_{k,i}(1/2), \cdots, Q_{k,i}^{(k)}(1/2)) \ge m - \left\lfloor \frac{i}{2} \right\rfloor. \tag{2.5}$$

Because of (2.3), (2.4) and (2.5), we get

$$S^-(P(0),\cdots,P^{(k)}(0)) = k . \tag{2.6}$$

Therefore, from Descaretes' rule, we know that the coefficients of the polynomial $Q_{k,i}$ strictly alternate in sign.

A similar discussion shows that (2.6) holds when k is odd. Thus, the following lemma follows.

<u>Lemma 2.</u> 1. $S^-(U_{k,2}^{(\ell)}) = k+\ell , \qquad \ell \in I_{k+1},$

2. The coefficients of the polynomial Q_{k+1} strictly alternate in sign.

By the method of construction of the sequence $\{U_{k,n}^{(i)}\}$ ((1.10), (1.11), we know

$$S^-(U_{k,n+1}^{(2\ell-1)}) = 2\,S^-(U_{k,n}^{(\ell)}),$$

$$S^-(U_{k,n+1}^{(2\ell)}) = 2\,S^-(U_{k,n}^{(\ell)}) + 1\,,$$

thus

$$S^-(U_{k,n}^{(\ell)}) = (k+1)2^{n-2} + \ell-1,$$

since this formula holds for $n = 2$, and follows for the general case by induction. Hence each function $U_{k,n}^{(\ell)}$ has one more sign-change than the preceding one. It is convenient to use the notation $U_{k,0}, U_{k,1}, \cdots$ instead of $U_{k,n}^{(\ell)}$ when we study their sign-changes. From now on, we would like to use both $\{U_{k,n}^{(\ell)}\}$ and $\{U_{k,n}\}$ freely with $U_{k,i} = U_i$ for $i < k$, obviously

$$U_{k,n}^{(\ell)} = U_{(k+1)2^{n-2}+\ell-1} \quad \text{for} \quad n \in Z\setminus\{0,1\}, \quad \ell \in I_{(k+1)2^{n-2}}. \tag{2.7}$$

<u>Theorem 4.</u> $S^-(U_{k,m}) = m\,, \quad m \in Z\,.$

I.e.

$$S^-(U_i) = i \quad , \quad i \in I_k \cup \{0\}$$

$$S^-(U_{k,n}^{(\ell)}) = (k+1)2^{n-2} + \ell-1, \qquad n \in Z\setminus\{0,1\}, \qquad \ell \in I_{(k+1)2^{n-2}}.$$

Now we begin to consider the convergence properties. The Fourier series of a given function F in terms of the functions $U_{k,i}$ is

$$F(x) \sim \sum_{i=0}^{\infty} \alpha_i\, U_{k,i}(x) \tag{2.8}$$

with

$$\alpha_i := \langle F(x),\ U_{k,i}(x)\rangle \ . \qquad (2.9)$$

Let

$$P_n F := \sum_{i=0}^{n} \alpha_i\, U_{k,i}(x)$$

be the n-th partial sum of the series (2.8).

Then P_nF is the best L_2-approximation to F from $M_n := \mathrm{span}(U_{k,i})_0^n$. Hence it is convergent to F if F is in L_2, since M_n is dense in L_2. We get the following theorem.

<u>Theorem 5.</u> If $f \in L_2[0,1]$, then $\lim_{n\to\infty} \|F - P_nF\|_2 = 0$.

Next we will prove that $P_{(k+1)2^{n-1}}F$ uniformly approximates $F \in L_\infty$.

It is well known [2] that

$$\|F - P_{(k+1)2^{n-1}}F\|_\infty \leq (1 + \|P_{(k+1)2^{n-1}}\|)\mathrm{dist}_\infty(F, M_{(k+1)2^{n-1}})$$

and we know

$$\|P_{(k+1)2^{n-1}}\| = \|P_k\| < \infty,$$

since least-square approximation for $M_{(k+1)2^{n-1}} = P_{k+1,\Delta_n}$ is local and $M_{(k+1)2^{n-1}}$ is dense in L_∞.

<u>Theorem 6.</u> Let $F \in C[0,1]$. $P_{(k+1)2^{n-1}}$ be L_2-projector onto $M_{(k+1)2^{n-1}}$ on $C[0,1]$, then

$$\lim_{n\to\infty} \|F - P_{(k+1)2^{n-1}}F\|_\infty = 0 \ .$$

But not every continuous function can be expanded in terms of the sequence U . We can prove that there exists a continuous function whose expansion in terms of the U's does not converge at a point of the interval.

The same kind of argument as in the proof of Theorem 7 in [3] shows that the following theorem holds.

Theorem 7. There exists a continuous function $f \in C[0,1]$ whose expansion

$$\sum_{i=0}^{n} \langle f(x), U_{k,i}(x)\rangle U_{k,i}$$

in terms of $\{U_{k,i}\}$ does not converge to f uniformly when $n \to \infty$.

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In this paper an orthonormal sequence of piecewise polynomials of degree k is given. We study the construction and sign-change properties of this sequence and consider the convergence of the corresponding Fourier series. The results generalize those obtained earlier for piecewise constant and piecewise linear functions.